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Monterey, California

LASER HEATING AND MIRROR DISTORTION

by

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This report presents the use of computer drawn graphs to facilitate solution of temperature profiles as a function of time. The solution of the one-dimensional unsteady heat-transfer equation for constant heat input due to laser radiation on one side of the sheet and insulation on the other was used to draw the graphs. Graphs have been drawn for three ranges of Fourier numbers: 0 to 1.0, 0 to 0.1, and 0 to 0.01. Sample problems are worked for three cases: temperature profiles within the metal plate as a function of time, time required to melt the front surface, and mirror distortion due to thermal expansion.

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time, time required to melt the front surface, and mirror distortion due to thermal expansion.

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NOMENCLATURE

α	thermal diffusivity, cm^2/sec
a	coefficient of linear thermal expansion, $1/^{o}K$
A	relative amplitude of electromagnetic radiation
b	y-coordinate of vertex of parabola
c	x-coordinate of vertex of parabola
d	thickness of metal sheet or mirror, cm
D	diameter of mirror, meters
f	ratio of aperture to focal length
F	focal length, meters
Fo	Fourier number, nondimensional time
I	intensity, watts/cm^2 or kW/cm^2
I_b	intensity of laser beam, watts/cm^2, kW/cm^2
k	thermal conductivity, watts/cm ^{o}K
ℓ	length of object expanding
$\bar{n}$	unit vector normal to surface
p	distance from focus to directrix, meters
r	reflectivity, or radial coordinate on mirror, meters
R	characteristic dimension of laser beam, or nondimensional mirror radius
t	temperature, ^{o}K
t_c	characteristic temperature, ^{o}K
t_0	initial temperature, ^{o}K
T	nondimensional temperature

θ time, seconds; or angle between beam and normal, degrees

x coordinate for distance through sheet of metal or mirror, centimeters, meters

X nondimensional distance through plate, x/d

I. INTRODUCTION

Among the applications of high energy lasers is welding. For this application and others, one would like an estimate of the time required to achieve a specified temperature. Other information that is useful is temperature distribution within the metal.

A set of graphs has been developed which permits rapid evaluation of time-to-temperature, temperature profiles, etc. First, the theory is given for the problem. Next, the limitations are outlined clearly. Finally, the graphs are discussed and sample problems are worked.

II. THEORY AND ANALYSIS

The time-dependent, one-dimensional equation for heat conduction is

$$\frac{\partial t}{\partial \theta} = \alpha \frac{\partial^2 t}{\partial x^2} \tag{1}$$

Equation (1) is solved for a plate of thickness d with a constant heat flux at the surface x = 0, i.e., the front wall. The back wall, x = d, is assumed to be insulated so that

$$\left.\frac{\partial t}{\partial x}\right|_{x=d} = 0 \tag{2}$$

The constant heat flux is due to a laser beam of intensity I, watts/cm^2. Consequently, the boundary condition on the front wall is

$$I = -k\frac{\partial t}{\partial x} \tag{3}$$

Introduce the following nondimensional variables into equations (1) and (3)

$$(a),\ X = x/d;\ (b),\ T = (t - t_0)/t_c;\ (c),\ t_c = Id/k;\ (d)\ Fo = \alpha\,\theta/d^2 \tag{4}$$

giving

$$\frac{\partial T}{\partial Fo} = \frac{\partial^2 T}{\partial X^2} \tag{5}$$

and

$$\frac{\partial T}{\partial X} = -1 \tag{6}$$

The solution to the problem formulated above is

$$T = Fo + \frac{1}{3} - (X - \frac{X^2}{2}) - \frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{e^{-Fo\pi^2 n^2}}{n^2}\cos n\pi X \tag{7}$$

Due to the exponential term in the series, for $Fo \geq 0.4$ the error in dropping the series is less than 1 per cent.

III. LIMITATIONS OF ANALYTICAL MODEL

The assumption of one-dimensional heating is valid so long as $R >> d$, where R is a characteristic dimension of the laser beam. The assumption of constant flux into the metal, as stated by equation (3), may or may not be realistic. This point will be discussed in more detail below.

Implicit in the equations (1) to (7) is the fact that material properties remain constant. Thermal conductivity, k, is a function of temperature. Likewise thermal diffusivity, α , is a function of temperature.

Return again to the question of constant heat flux due to laser radiation. The laser beam has the intensity I_b. The value of intensity that is correct for equation (3) is

$$I = I_b \cos\theta(1 - r) \tag{8}$$

Equation (8) accounts for the spreading of the radiation due to angle θ (See Fig. 1.) and the reflectivity. The reflectivity, r, is a weak function of temperature.

The charts are of value only up to the melting temperature. All parameters in the problem vary rapidly with temperature as phase changes from solid to liquid. Furthermore, the heat of vaporization is not incorporated in the model. When the surface becomes molten, vapor forms. The vapor may be absorbing; as a consequence, the intensity of radiation may be greatly reduced.

In summary, the model provides adequate answers up to the melting temperature if the radiant heat flux, I, is correctly determined by equation (7).

IV. DISCUSSION OF GRAPHS

Figures 2 to 4 are graphs which have been calculated using equation (7) and the appropriate nondimensional variables, equations (4a) to (4d), which were defined earlier. Note that there are three graphs, i.e., one in upper left hand (ULH) corner, one in upper right hand (URH) corner, and one in lower right hand (LRH) corner. The primary graph is in the URH corner; both ULH and LRH graphs are for convenience in the use of the primary graph. The URH graph is a plot of nondimensional temperature, T, in terms of nondimensional time, Fo. The Fourier number is essentially a nondimensional time. To use the graphs, one must know the intensity, I; the thickness of the plate, d; the thermal conductivity, k; and thermal diffusivity, α. A characteristic temperature, t_c, is calculated using equation (4c); t_c is used to enter the ULH graph.

The LRH graph has been drawn for aluminum. It is a plot of

$$d = \sqrt{\frac{\alpha \theta}{Fo}} \qquad (9)$$

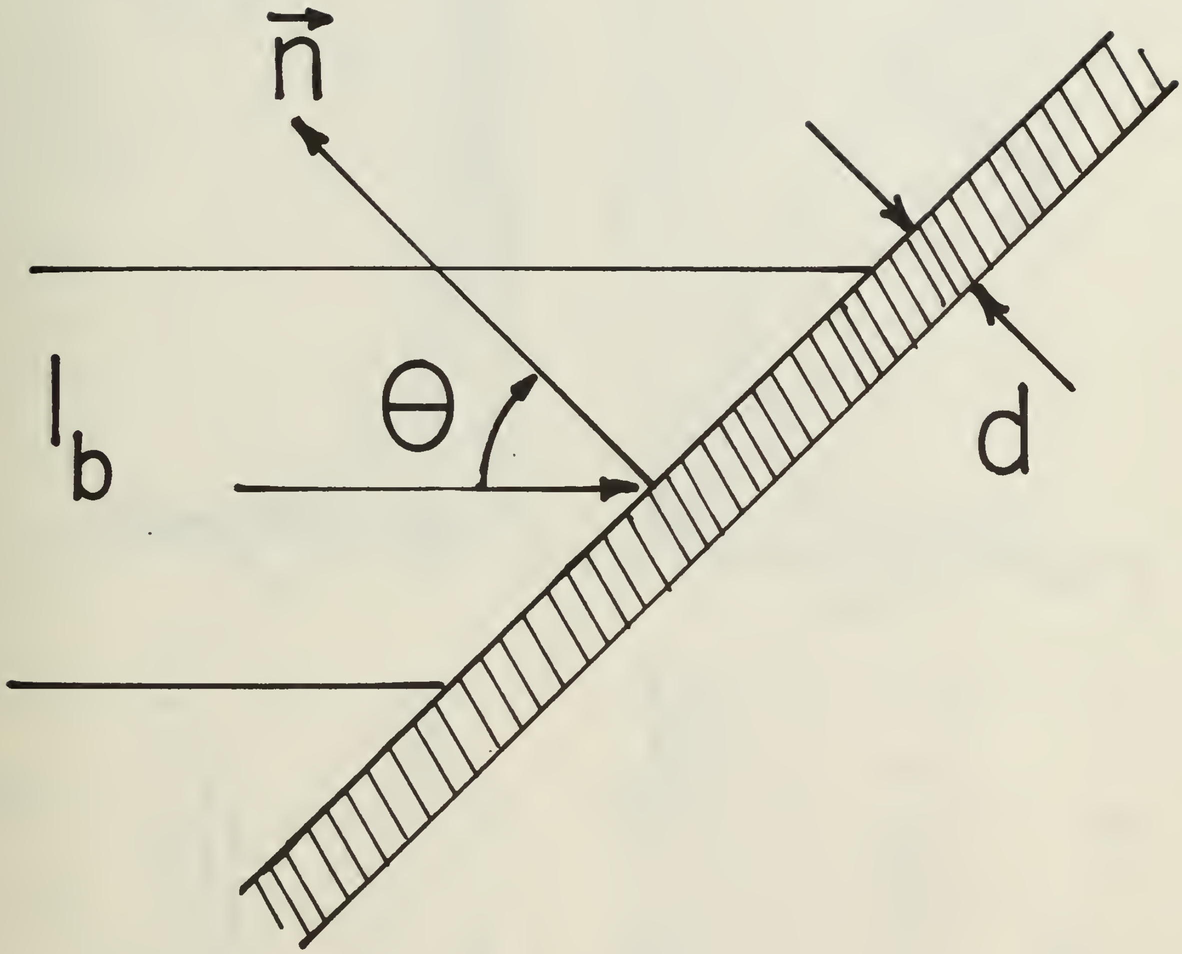

Figure 1. Geometry of Heating by Laser Radiation.

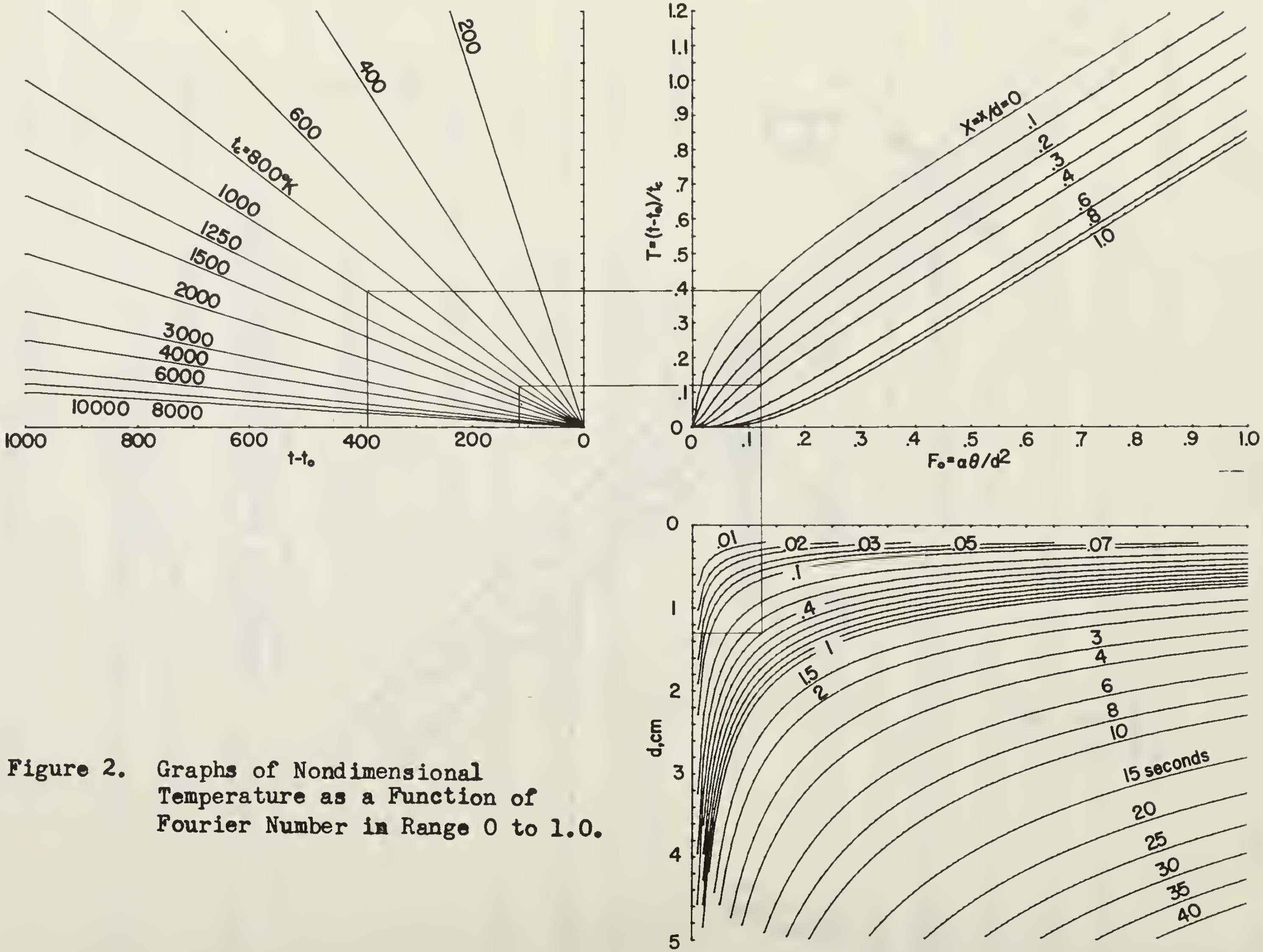

Figure 2. Graphs of Nondimensional Temperature as a Function of Fourier Number in Range 0 to 1.0.

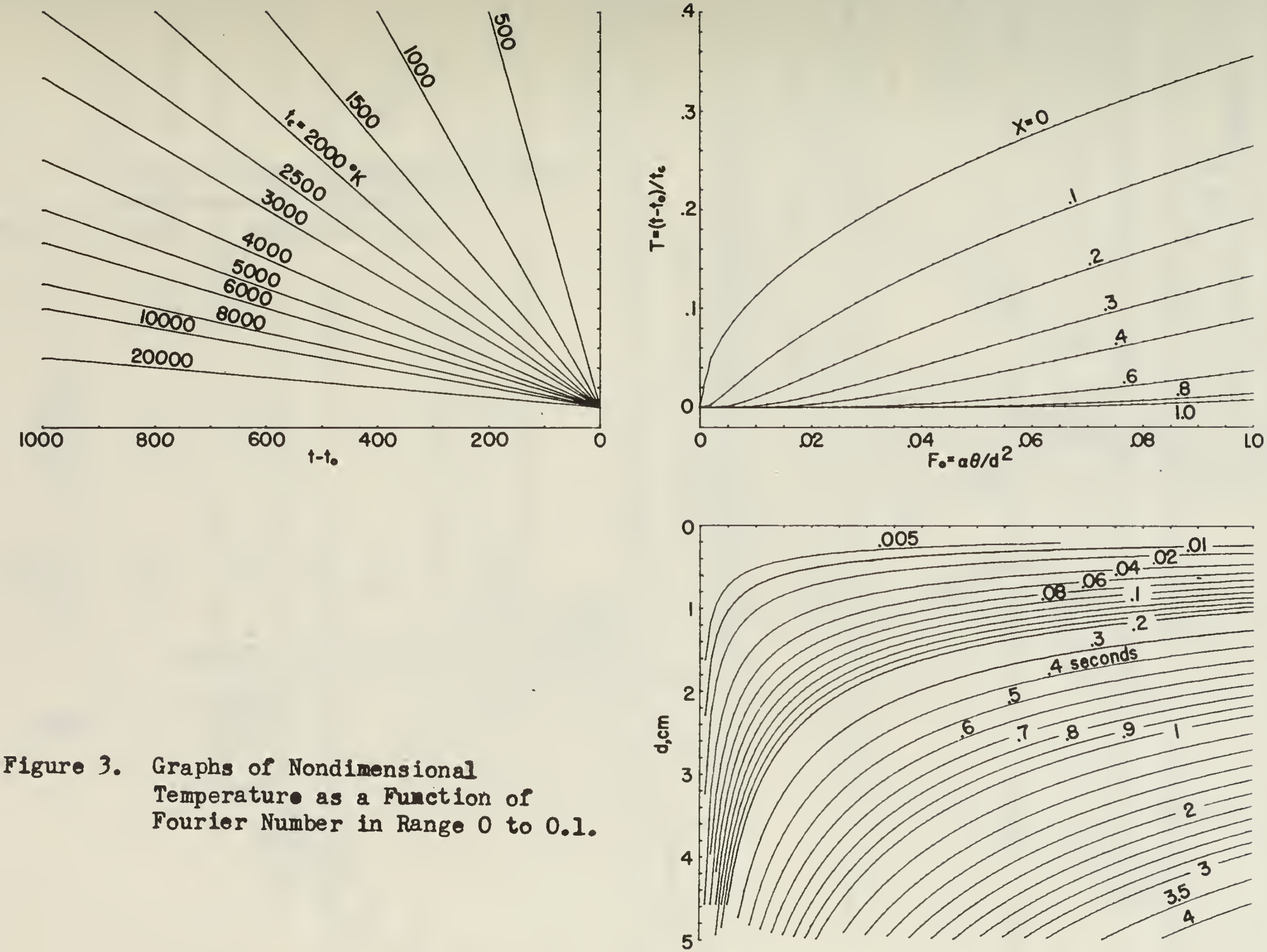

Figure 3. Graphs of Nondimensional Temperature as a Function of Fourier Number in Range 0 to 0.1.

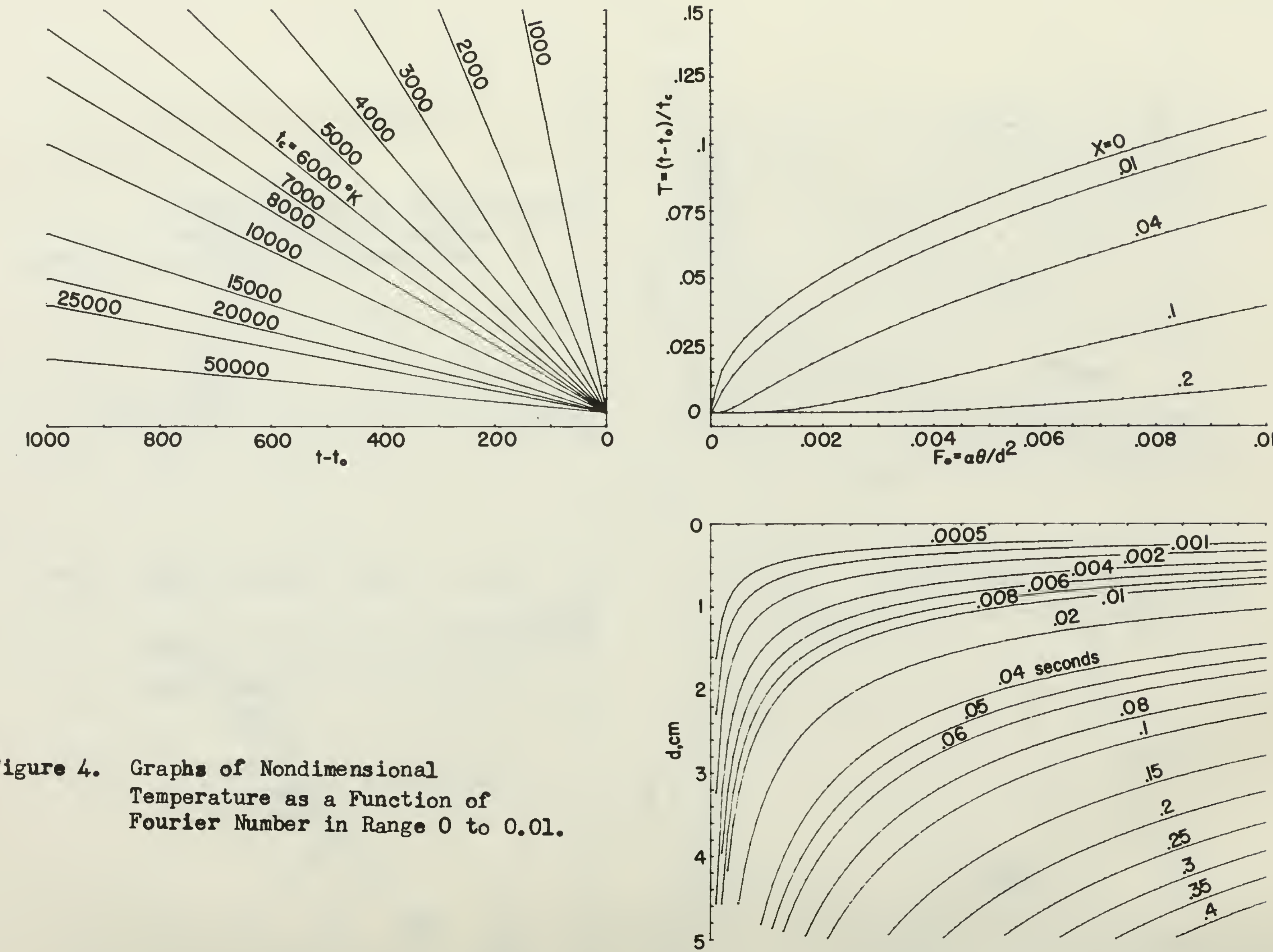

Figure 4. Graphs of Nondimensional Temperature as a Function of Fourier Number in Range 0 to 0.01.

For aluminum, the thermal diffusivity is 0.521 cm^2/sec. The time, θ, is used as a parameter in the LRH graph.

Both the ULH and URH graphs are universal; i.e., they can be applied to any homogeneous substance. However, the LRH graph is applicable only for aluminum. A simple calculation can be used to convert the time, θ, read from the graph to the time, θ', for other materials. This calculation is based on equation (9) as follows:

$$\theta' = \theta\alpha / \alpha'$$

The other material has thermal diffusivity, α'. For example, suppose $\alpha' = 0.2605\ cm^2/sec$ for material "X"; hence

$$\theta' = \frac{(1.2\ \text{second})(0.521\ cm^2/sec)}{(0.2605\ cm^2/sec)} = 2.4\ \text{second}$$

With the simple calculation outlined above, the graphs of Figs. 2, 3, and 4 are universal.

Some sample problems will now be discussed.

A. TEMPERATURE AT A GIVEN TIME AND LOCATION

Sample Problem 1

Assume a plate thickness of d = 1.3 cm and an initial temperature of $t_0 = 300^\circ K$. Assume I = 1000 $watt/cm^2$ and k = 1.3 $watt/cm^\circ K$. If time $\theta = 0.4$ second has elapsed since the laser beam was switched on, what is the temperature at front face?

Calculate t_c.

$$t_c = \frac{(1000)(1.3)}{(1.3)} = 1000^\circ K$$

Enter the LRH graph with $d = 1.3$ cm. Move to the right to the curve for $\theta = 0.4$. Then move upward to the curve for $X = 0$, which is the front face. Note that $Fo = 0.12$ and $T = .39$. Move toward the left until the line for $t_c = 1000^{\circ}K$ is intersected. Then move down to the axis and read $t - t_0 = 390$. Since the initial temperature was $300^{\circ}K$, the front face has a temperature $t = 690^{\circ}K$.

Sample Problem 2

Sample Problem 2 is the same as Problem 1 except that the temperature at $x = 0.52$ cm is desired. Calculate $X = x/d = 0.52/1.3 = 0.4$. Where the vertical line running between the LRH and URH graphs intersects $X = 0.4$ curve, draw a horizontal line. At the intersection with the $t_c = 1000^{\circ}K$ curve, descend vertically to the axis. Read $t - t_0 = 115$. At a distance $x = 0.52$ cm and a time $\theta = 0.4$ sec, the temperature is $415^{\circ}K$.

B. TIME TO MELT FRONT SURFACE

Sample Problem 3

The melting temperature of aluminum is $933^{\circ}K$. Assume $t_0 = 333^{\circ}K$. If a beam has an intensity of 2000 watt/cm^2, how much time is required to begin melting the front surface of an aluminum plate with thickness 0.65 cm?

Calculate t_c.

$$t_c = \frac{(2000)(0.65)}{1.3} = 4000^{\circ}K$$

Subtracting $333^{\circ}K$ from $933^{\circ}K$, one enters the ULH graph at $600^{\circ}K$ and moves upward to $t_c = 4000^{\circ}K$ curve. Moving horizontally, one crosses the T-axis at 0.152 and stops at the intersection of $X = 0$ curve. Then a line is dropped vertically crossing the Fourier number axis at 0.018. Draw a horizontal curve in the LRH graph at $d = 0.65$ cm. The intersection of the horizontal and vertical lines shows that $\theta = .015$ second. Note that the front face is ready to melt while the temperature of the back face ($X = 1$) remains $t_0 = 333^{\circ}K$.

V. MIRROR DISTORTION DUE TO RADIANT HEATING

High powered lasers often use metal mirrors which undergo thermal expansion and optical distortion due to absorption of radiation. Depending on mirror geometry, complex, three-dimensional calculations may be required. In this section, a sample calculation for determining one-dimensional mirror distortion is illustrated.

The formula for thermal one-dimensional strain is

$$\frac{\Delta \ell}{\ell} = a \Delta t \tag{10}$$

where a is the coefficient of linear thermal expansion, ℓ is the length of the object expanding, $\Delta \ell$ is the increase in its length, and Δt is the temperature change involved.

The linear thermal expansion coefficient, a, is a function of temperature. Table I gives its values for different temperatures of aluminum. Values for other metals as a function of temperature can be found in the American Institute of Physics Handbook [Ref. 1].

Table I. Temperature Variation of Linear Thermal Expansion Coefficient for Aluminum [Ref. 1].

t (^{o}K)	a ($10^{-6}/^{o}$K)
300	23.2
350	24.1
400	24.9
500	26.4
600	28.3
700	30.7

The expansion is given by equation (11).

$$\frac{\Delta \ell}{d} = \int_0^d a(t)(t - t_0)dx \tag{11}$$

where x is the coordinate for the distance through the mirror and d is the mirror thickness. For this sample problem, numerical evaluation has been chosen. The following characteristics for the mirror were used: parabolic mirror, mirror radius = 0.05 m, f number = 1.0, intensity of beam = 40 kW cm^2, mirror absorption = 0.01, and Fresnel number = 10.

Using the facts that the mirror is parabolic with a f number of 1.0 and thickness at the center of 2 cm, the thickness of the mirror at other radial points can be calculated.

$$f = \frac{\text{Focal length}}{\text{Diameter}} = \frac{F}{D}$$

$$F = Df = (0.1\ \text{m})(1) = 0.1\ \text{m}$$

The standard form for a parabola is

$$2p(y - b) = (r - c)^2 \tag{12}$$

where p is the distance from the directrix to the focus of the parabola and (c,b) are the coordinates of its vertex. Thus p/2 is the focal length of the mirror. So p/2 = 0.1 m, and p = 0.2 m. Putting the vertex (c,b) of the parabola at the origin yields c = b = 0. Therefore, the equation of the parabola is $2(0.2)(y - 0) = (r - 0)^2$ or $y = 2.5r^2$. Solving this equation for different values of r gives the increase in the thickness of the mirror at different radii. The actual thickness can be found by adding 2 cm, which is the thickness at the center of the mirror. The thickness of the mirror at different radial locations is given in Table II.

Table II. Mirror Thickness, Intensity of Absorbed Power, and Characteristic Temperature

R	r, m	d, cm	Intensity, W/cm^2	t_c, ^oK
0	0.0	2.000	400	615
0.3	0.015	2.056	296	468
0.7	0.035	2.306	77	137
1.0	0.05	2.625	5	9.77

To find the intensity of the laser radiation at various radii on the mirror, the amplitude profile from Figure 3.11 from Lengyel [Ref. 2] was used. The absorbed intensity at the center of the mirror was assumed to be 400 watts/cm^2 which was obtained by multiplying radiation intensity by mirror absorption ($40kW/cm^2 \times 0.01$). Amplitude and intensity are related by

$$\frac{I_1}{I_0} = \left[\frac{A_1}{A_0}\right]^2 \tag{13}$$

where A_0 and A_1 are the amplitudes at two different points (from the referenced graph), and I_0 is the known intensity at the mirror axis. The intensities also are tabulated in Table II.

In this problem, one-dimensional heating is assumed; i.e., the effects of heat transfer in the radial direction are ignored. To solve the problem, it is necessary to calculate t_c. This can be done using equation (4c) where $k = 1.3$ watt/cm ^oK. The values of t_c for different values of d are listed in Table II.

To evaluate the integral numerically, the mirror is imagined as being divided into slabs. The expansion of each of these slabs is calculated based on the mean slab temperature. These calculations can be made using the set of graphs, Figs. 2 to 4, explained earlier in this paper. The calculation methods become apparent by referring to Table III which summarizes the results for the centerline position on the mirror.

The graph, Fig. 5, is a profile of the thermal expansion. From the graph, it can be seen that considerable distortion will occur, even at times as small as 0.3 sec. Modest quality optics have a tolerance less than one quarter of the wavelength of the light. The distortion in this problem would be considerably more than a quarter wavelength.

VI. CONCLUSIONS

Graphs have been developed for the solution of many types of problems involved with laser heating. Rapid solutions become possible by suitable use of the figures. Sample problems were presented for temperature profiles within the plate, time to melt the front surface, and mirror distortion due to thermal expansion.

Table III. Summary of Calculations for Thermal Expansion at Mirror Center

$X = x/d$	x, cm	$t - t_0$*	$\langle t - t_0 \rangle$	a	Δx	δx, microns
0	0	140				
			110	.252	.2	5.544
0.1	0.2	80				
			65	.244	.2	3.172
0.2	0.4	50				
			37.5	.239	.2	1.793
0.3	0.6	25				
			17.5	.235	.2	0.823
0.4	0.8	10				
			6	.233	.4	0.559
0.6	1.2	2				
			1	.232	.4	0.093
0.8	1.6	0				
			0	-	-	-
1.0	2.0	0				
		Total Thermal Expansion, $\Delta \ell$				11.983 microns

*Obtained from Fig. 3 with d = 2 cm and $t_c = 615^{\circ}$K.

$\langle t - t_0 \rangle$ average slab temperature in excess of initial temperature t_0, $^{\circ}$K.

a coefficient of linear thermal expansion, microns/cm $^{\circ}$K.

Δx slab thickness, cm.

δx thermal expansion of slab, microns.

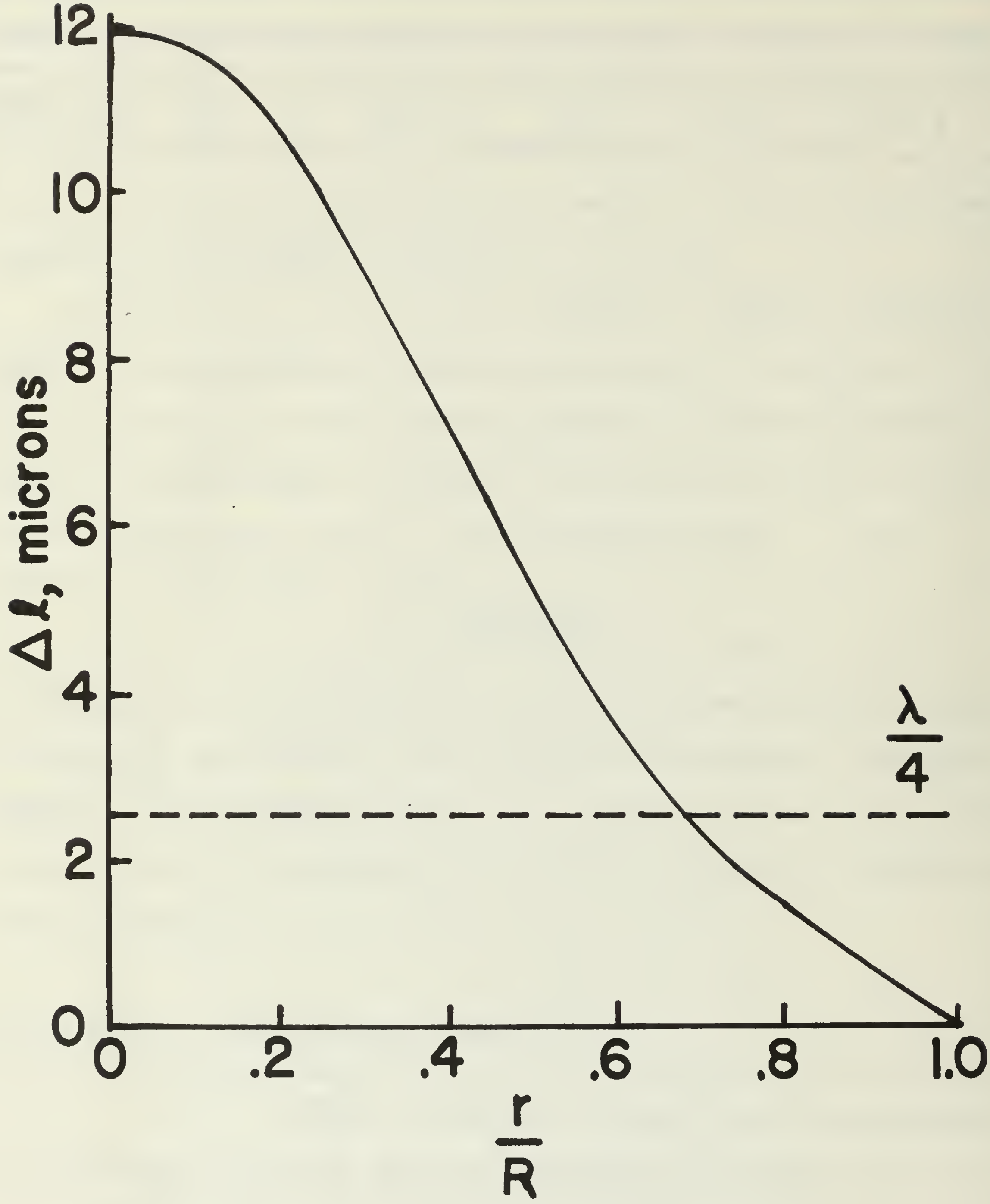

Figure 5. Profile of Thermal Expansion at 300 Millisecs.

BIBLIOGRAPHY

1. Gray, D. E., Editor, American Institute of Physics Handbook (New York: McGraw Hill, 1963).

2. Lengyel, Bela A., Lasers, Second Edition (New York: John Wiley and Sons, 1971).

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